



Uncertainty in Software Models

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(part of joint work with more people)



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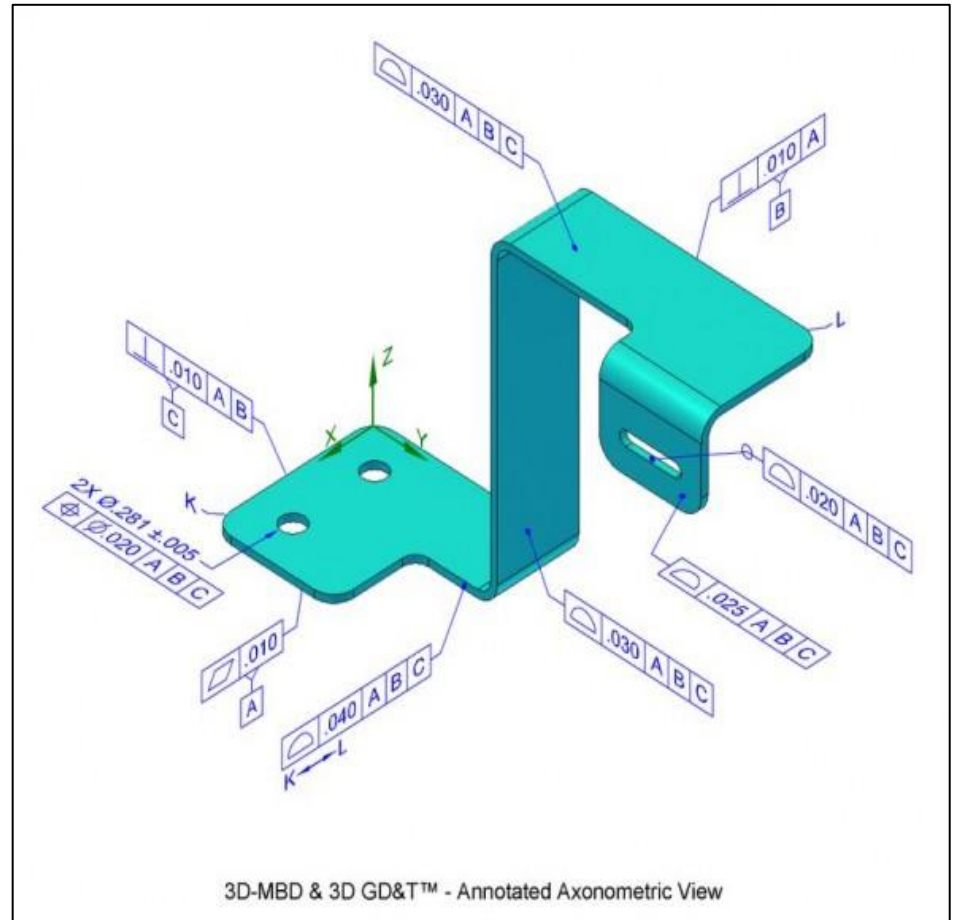
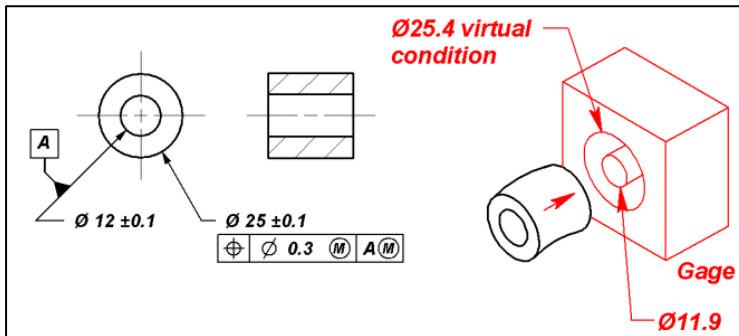
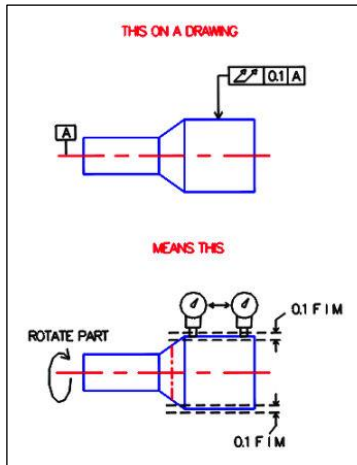
Uncertainty

Uncertainty: Quality or state that involves imperfect and/or unknown information

- It applies to: predictions of future events, estimations, physical measurements, or properties of a system, its elements or its environment
- due to:
 - Underspecification of the problem or solution domains
 - Lack of knowledge of the system, its environment, or its underlying physics
 - Lack of precision in measurements
 - Imperfect, incorrect, or missing information
 - Numerical approximations
 - Values and parameters indeterminacy
 - Different interpretations of the same evidences by separate parties

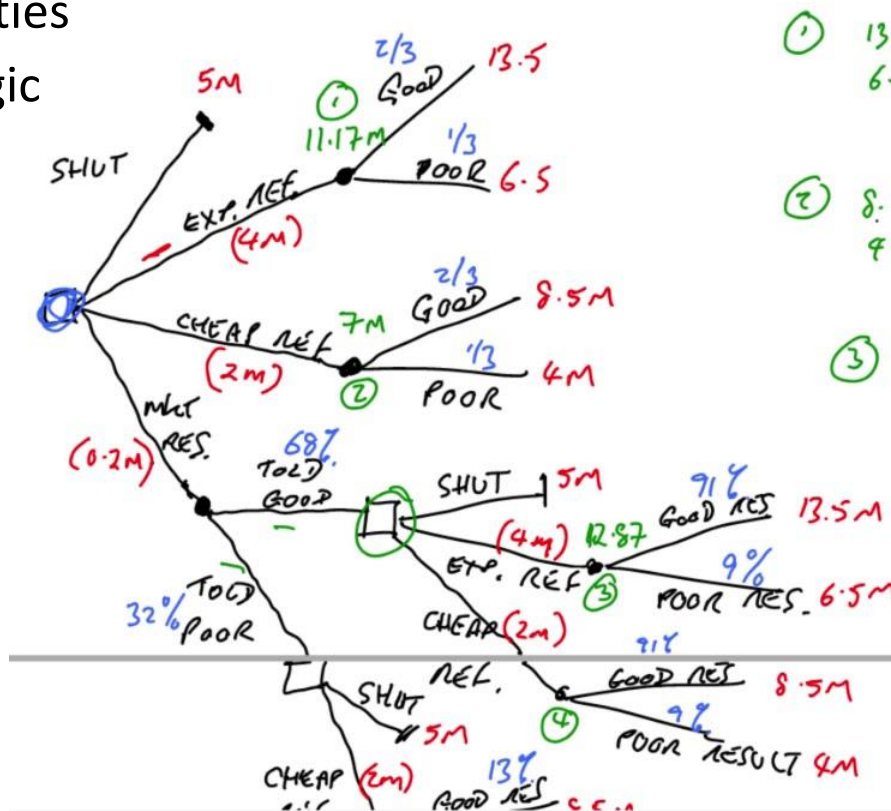
“There is nothing certain, but the uncertain” (*proverb*)

Uncertainty in mechanical systems



Many different formalisms and theories to quantify uncertainty

- Bayesian Belief Networks (BBN)
- Monte Carlo simulations
- Decision theory/trees
- Probabilities
- Fuzzy Logic
- ...



$$\textcircled{1} \begin{array}{l} 13.5 \times \frac{2}{3} = 9 \\ 6.5 \times \frac{1}{3} = 2.17 \\ \hline 11.17 \end{array}$$

$$\textcircled{2} \begin{array}{l} 8.5 \times \frac{2}{3} = 5.67 \\ 4 \times \frac{1}{3} = 1.33 \\ \hline 7.00 \end{array}$$

$$\textcircled{3} \begin{array}{l} 13.5 \times 0.91 = 12.29 \\ 6.5 \times 0.09 = 0.585 \\ \hline 12.875 \end{array}$$

$$\textcircled{4} \begin{array}{l} 8.5 \times 0.91 = 7.735 \\ 4 \times 0.09 = 0.36 \\ \hline 8.095 \end{array}$$



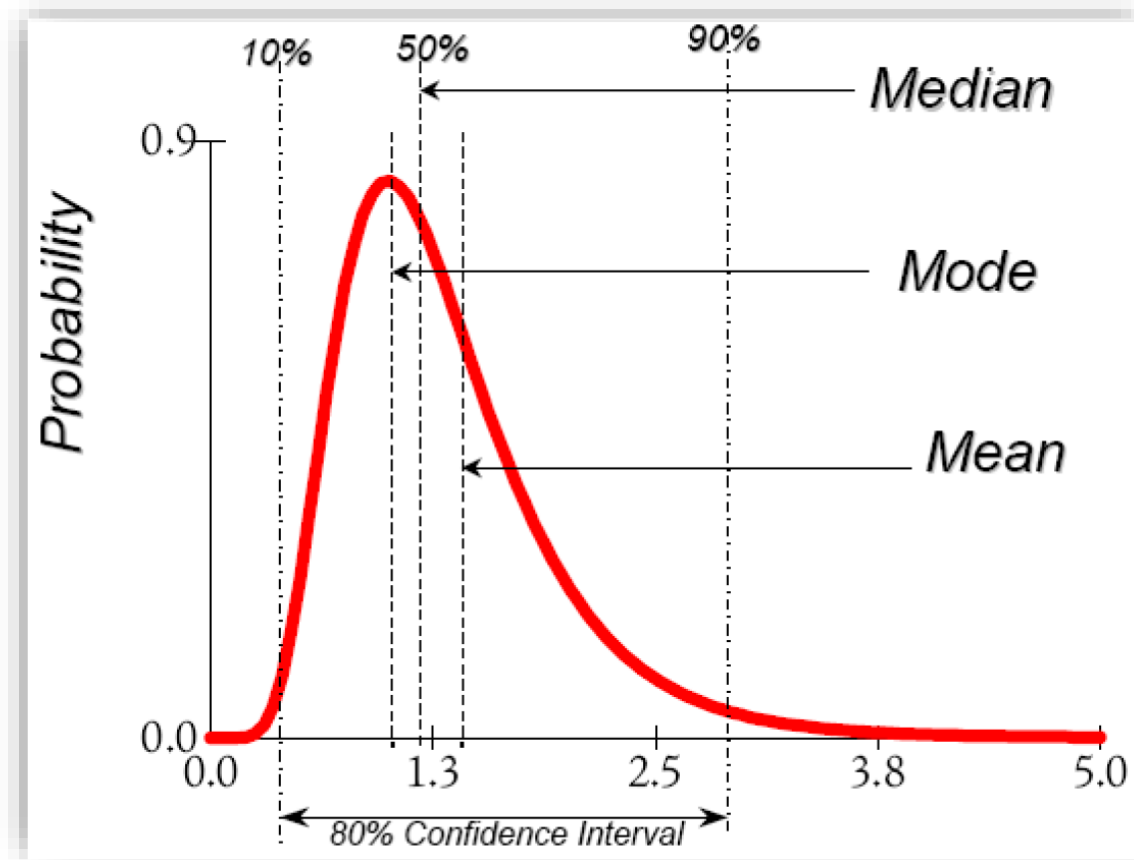
A classification of uncertainty (according to its nature)

- **Aleatory** Uncertainty – A kind of uncertainty that refers to the *inherent* uncertainty due to the *probabilistic variability or randomness* of a phenomenon
 - Examples: measuring the speed of a car, or the duration of a software development process
 - This type of uncertainty is **irreducible**, in that there will always be variability in the underlying variables.
- **Epistemic** Uncertainty – A kind of uncertainty that refers to *the lack of knowledge* we may have about the system (modeled or real).
 - Examples: Ambiguous or imprecise requirements about the expected system functionality, its envisioned operating environment, etc.
 - This type of uncertainty is **reducible**, in that additional information or knowledge may reduce it.

Types of uncertainty (according to their sources)

- **Measurement uncertainty:** A kind of *aleatory* uncertainty that refers to a set of possible states or outcomes of a measurement, where probabilities are assigned to each possible state or outcome
- **Occurrence uncertainty:** a kind of *epistemic* uncertainty that refers to the degree of belief that we have on the actual existence of an entity, i.e., the real entity that a model element represents
- **Belief uncertainty:** A kind of *epistemic* uncertainty in which a *belief agent* is uncertain about any of the *statements* made about the system or its environment.
- **Design uncertainty:** A kind of *epistemic* uncertainty that refers to a set of possible design decisions or options, where probabilities are assigned to each decision or option
- **Environment uncertainty:** lack of certainty about the surroundings, boundaries and usages of a system and of its elements
- **Spatiotemporal uncertainty:** lack of certainty about the geographical or physical location of a system, its elements or its environment, or about the time properties expressed in a statement about the system or its environment

Measurement Uncertainty



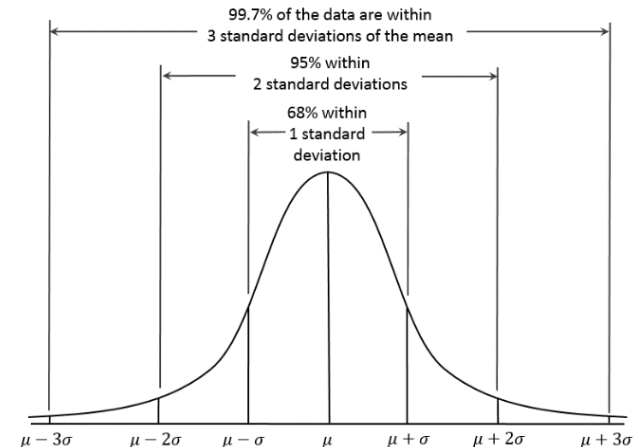
Measurement uncertainty

- **Measurement uncertainty:** A kind of *aleatory* uncertainty that refers to a set of possible states or outcomes of a measurement
- Normally expressed by a parameter, associated with the result of a measurement x , that characterizes the dispersion of the values that could reasonably be attributed to the measurand: **the standard deviation u of the possible variation of the values of x**
- Representation: $x \pm u$ or (x, u)

- **Examples:**

- Normal distribution: (x, σ) with mean x , and standard deviation σ
- Interval $[a, b]$: Uniform distribution is assumed

$$(x, u) \text{ with } x = \frac{a+b}{2}, u = \frac{(b-a)}{2\sqrt{3}}$$



Measurement uncertainty

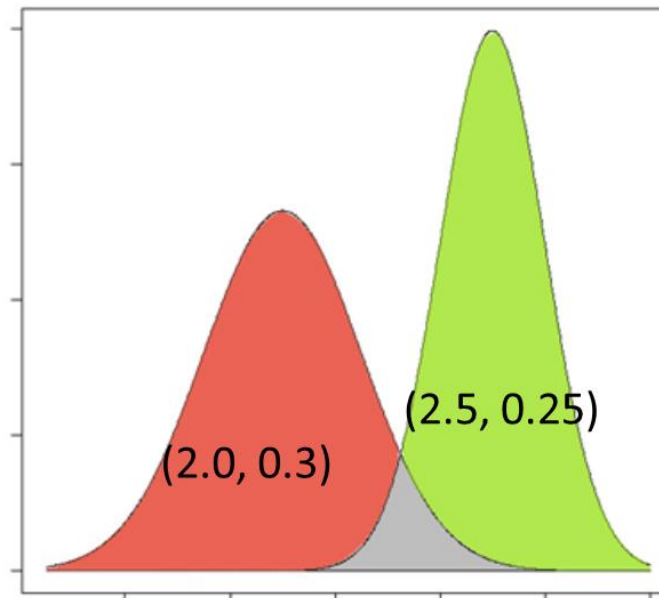
- However, the situation is not the same in software models



RoundObject
+posX : Real +posY : Real +posZ : Real +weight : Real +width : Real +height : Real
+move(dX : Real, dy : Real, dz : Real) +catch() +drop() +fitsIn(other : RoundObject)

Some problems with Measurement Uncertainty

- Computations with uncertain values must respect the *propagation of uncertainty* (uncertainty analysis)
 - In general, this is a complex problem, which cannot be manually managed
- *Comparison of uncertain values* is no longer a Boolean property!
 - How to compare 17.7 ± 0.2 with 17.8 ± 0.2 ?
- Other primitive datatypes are also affected by uncertainty
 - Strings (OCR)
 - Enumerations
 - Collections



UBoolean

- UBooleans are pairs (b, c)
 - where $b:\text{Boolean}$ and $c:\text{Real}$, $c \in [0, 1]$
 - c represents the confidence that the actual value of the value is indeed b
 - Canonical form: (true, c)
 - Equivalence relation: $(b, c) = (\text{not } b, 1 - c)$
- Operations
 - Redefined basic operations: and, or, not
 - Redefined secondary operations: implies, equivalent, xor
 - Conversion operations: `toBoolean()` and `toBooleanC(c:Real)`

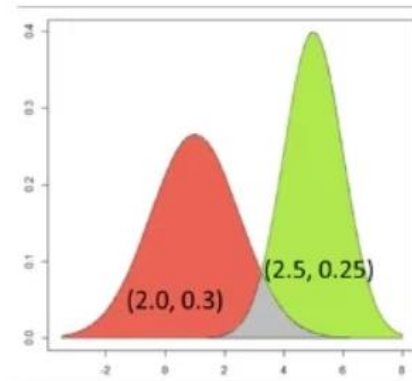
Primitive datatypes extended with Uncertainty

- Extended primitive datatypes
 - Real → UReal UReal(17.8,0.2) \equiv 17.8 \pm 0.2
 - Boolean → UBoolean UBoolean(true, 0.8)
 - String → Ustring UString("Implementaci6n",0.93)
 - Enum → UEnum UColor{ (#red,.9), (#orange,0.09), (#purple,0.01) }
- An algebra of operations on uncertain datatypes extending OCL/UML types
- Operations are **closed** in this algebra and automatically **propagate** uncertainty

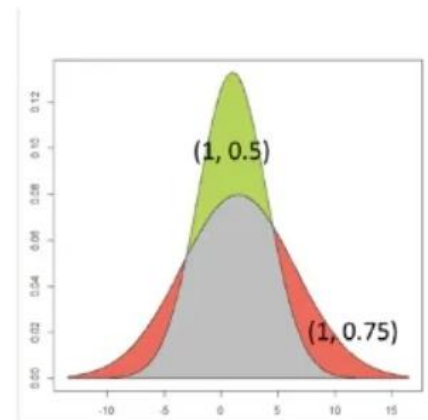
```
use> ?UReal(17.7,0.2) < UReal(17.8,0.2)
-> UBoolean(true, 0.1974125487) : UBoolean

use> ?UReal(17.7,0.2) = UReal(17.8,0.2)
-> UBoolean(true, 0.8025874513) : Uboolean

use> ?UReal(17.7,0.2) > UReal(17.8,0.2)
-> UBoolean(true, 0.0) : Uboolean
```

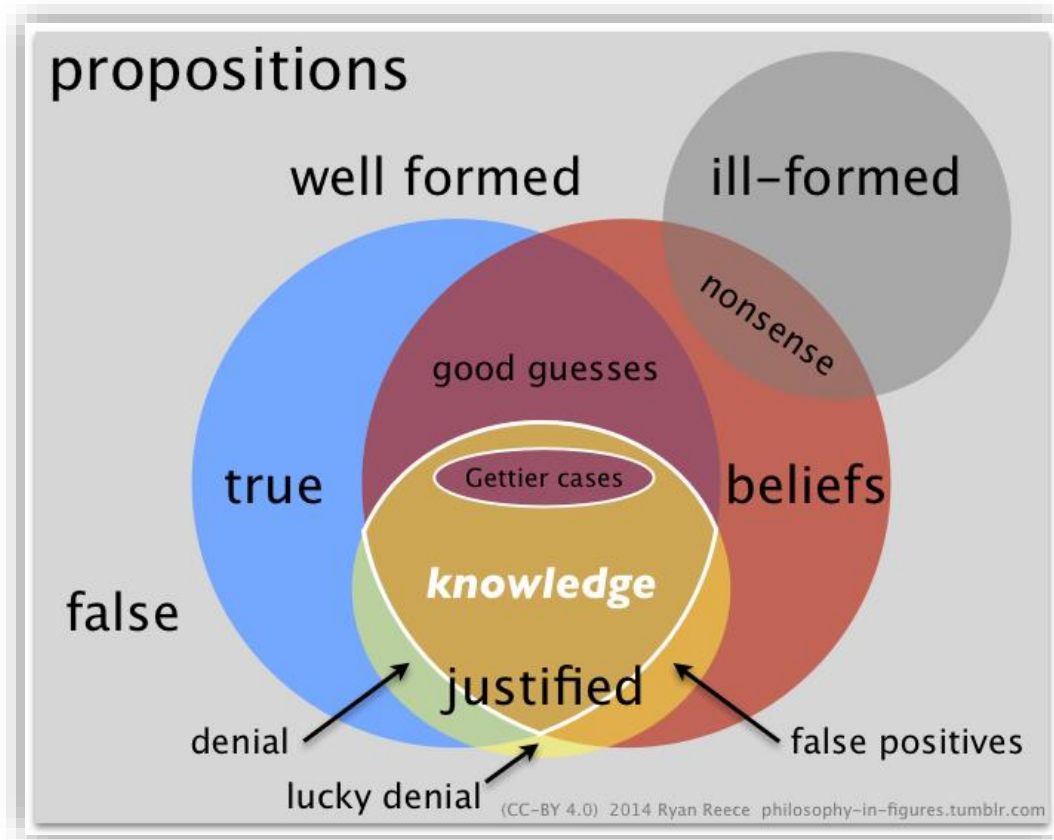


(0.893, 0.106, 1.11 · 10⁻¹⁶)



(0.152, 0.754, 0.094)

Belief Uncertainty



Belief uncertainty

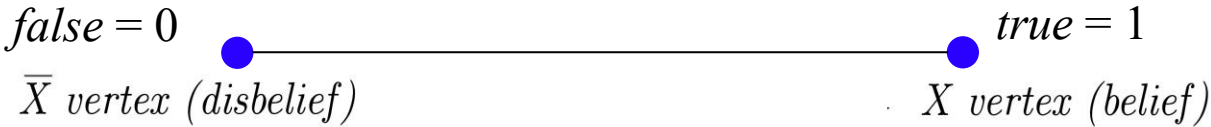
- **Belief uncertainty:** A kind of *epistemic* uncertainty in which the modeler, or any other *belief agent*, is uncertain about any of the *statements* made about the system or its environment.
 - By nature, it is always *subjective*
 - *It may not always be possible to determine whether or not a belief statement is valid.*
 - A belief statement may not necessarily correspond to objective reality.
 - This means that it could be completely false, or only partially true, or completely true.
 - The validity of a statement may only be *meaningfully defined within a given context or purpose.*
 - Thus, the statement that “the Earth can be represented as a perfect sphere” may be perfectly valid for some purposes but invalid or only partly valid for others.
- **Belief agent:** An entity (human, institution, even a machine) that holds one or more beliefs
- **Belief statement:** *Statement* qualified by a *degree of belief*
- **Degree of belief:** Confidence assigned to a statement by a belief agent. Normally expressed by quantitative or qualitative methods (e.g., a grade or a probability “*credence*”)

Classical Boolean Logic

$false = 0$ ●
 \bar{X} vertex (disbelief)

● $true = 1$
 X vertex (belief)

Probabilistic Logic



Kleene (three-valued) Logic

Uncertainty



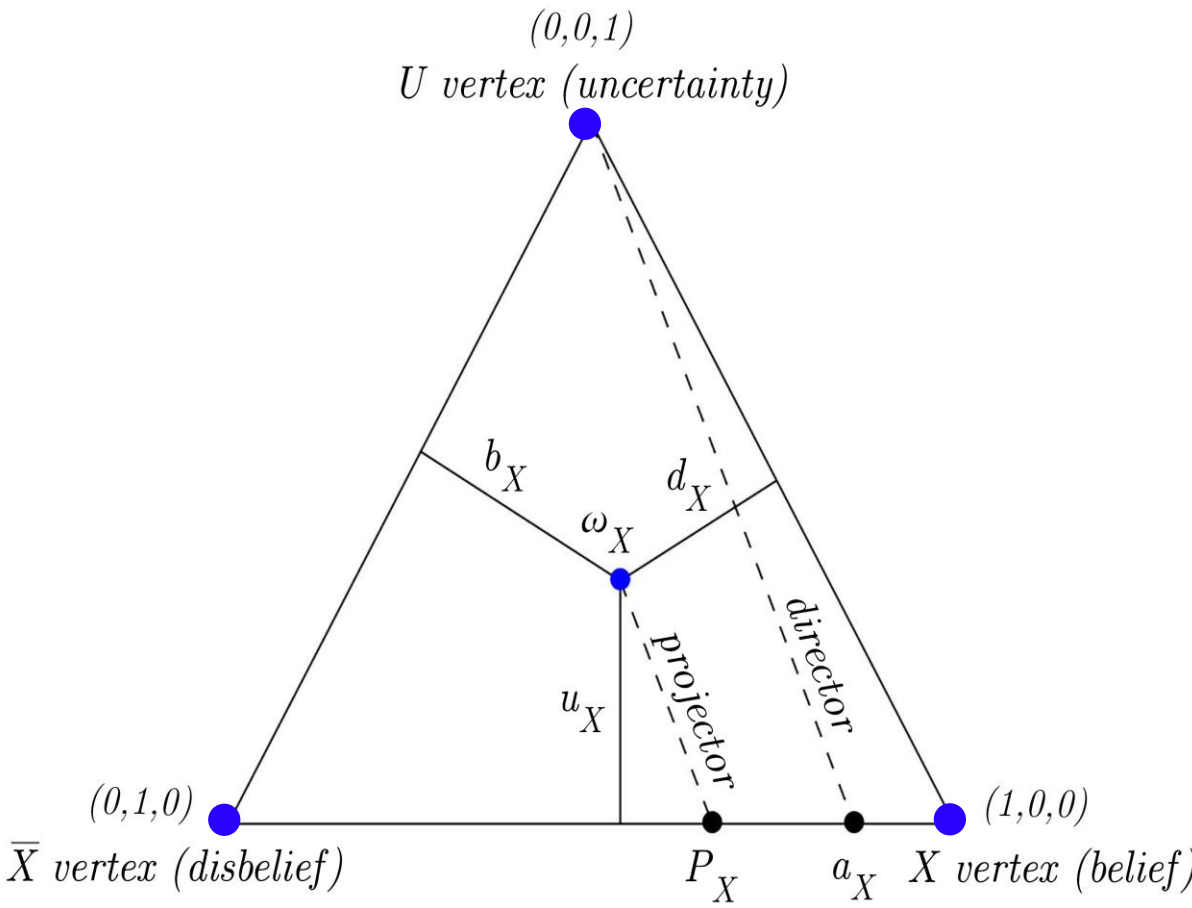
\bar{X} vertex (disbelief)



X vertex (belief)



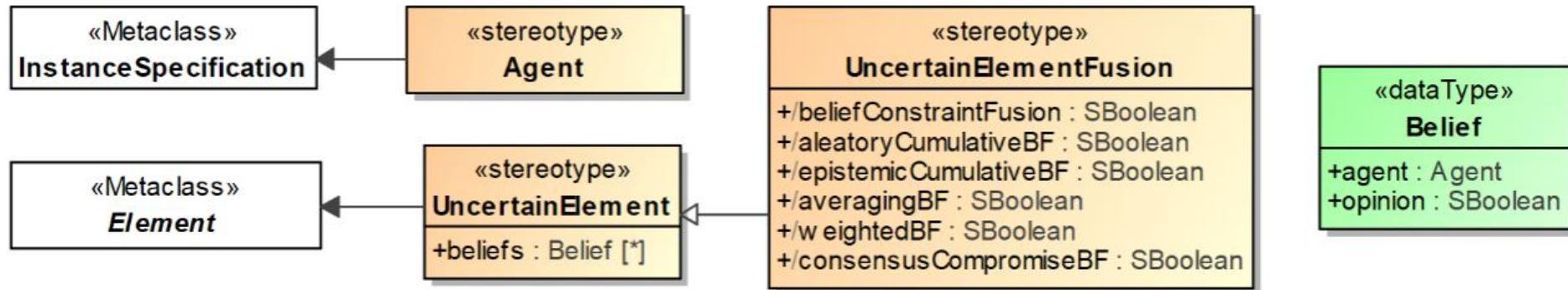
Subjective Logic



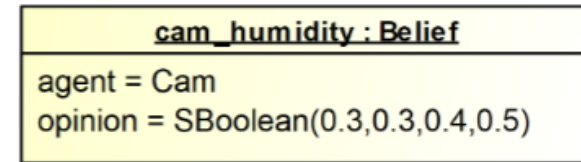
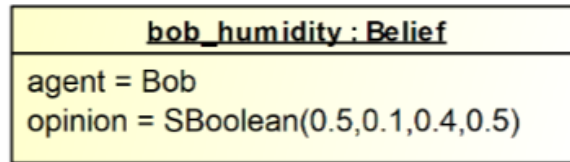
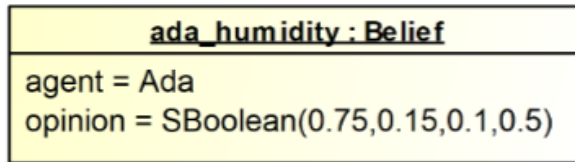
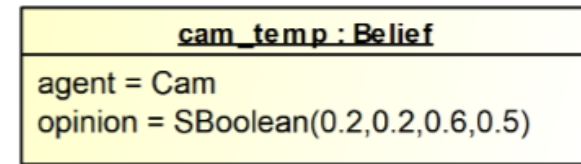
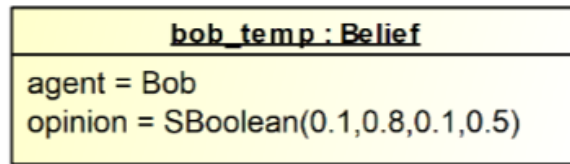
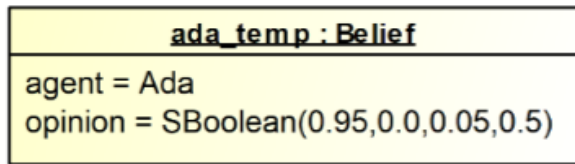
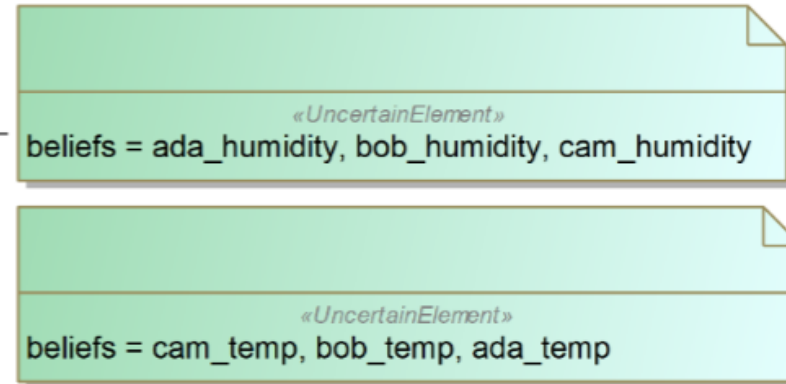
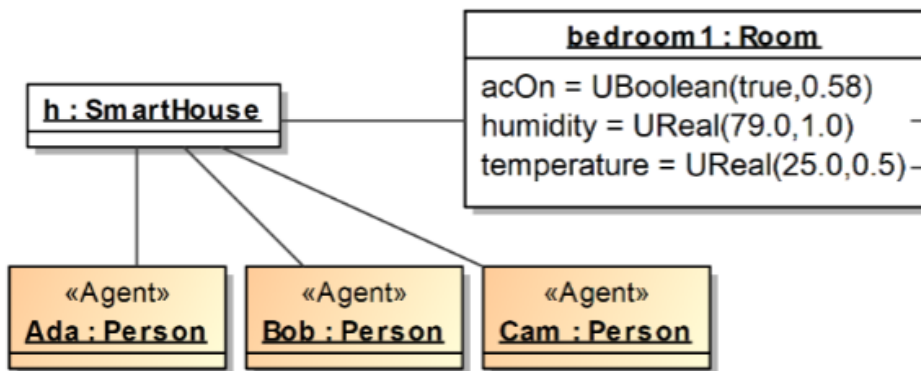
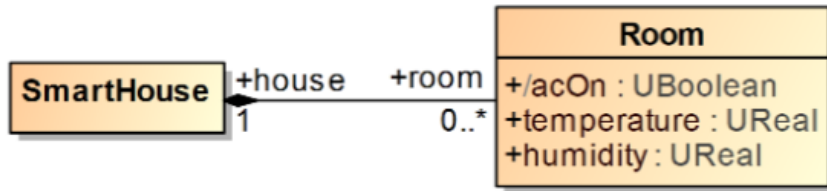
Subjective logic in UML/OCL

- $SBoolean(b, d, u, a)$
 - b represents the degree of **belief** that the agent has about the statement
 - d represents the degree of **disbelief**
 - u represents the **uncertainty** that the agent expressing the opinion has about the statement, i.e., the degree of trust
 - a is the (objective) **prior probability** assigned to the statement, based on previous evidence (also called “**base rate**”).
- $b + d + u = 1$
- Boolean values lifted to SBoolean:
 - $true \equiv SBoolean(1, 0, 0, 1)$
 - $false \equiv SBoolean(0, 1, 0, 0)$
- UBoolean values lifted to SBoolean:
 - $Uboolean(true, c) = SBoolean(c, 1-c, 0, c)$
- Projection from SBoolean to UBoolean:
 - $SBoolean(b, d, u, a).projection() = UBoolean(true, b + u*a)$

Belief Profile



Belief Profile



Fusion Operators

- Merge the beliefs of different agents on the same model elements

	Willingness to compromise	Assumed dependency between opinions	Vacuous opinion as neutral element	Preserve shared beliefs, and conflicting opinions are turned into vague belief
Belief Constraint Fusion (BCF)	NO. If conflict, undefined result.	Only for fusion of agents' preferences	YES	NO
Cumulative Belief Fusion (CBF)	YES	NO	YES	NO
Averaging Belief Fusion (ABF)	YES	YES	NO	NO
Weighted Belief Fusion (WBF)	YES	YES	YES	NO
Consensus and Compromise Fusion (CCF)	YES	YES	YES	YES

^[1]Not applicable for fusing evidence from different agents to determine the most likely hypothesis or actual event.

^[2] There are two types of Cumulative Belief Fusion operators: Epistemic Cumulative Fusion (E-CBF) and Aleatory Cumulative Fusion (A-CBF). Their use depends on the nature of the fused opinions.

Fusion Operators

Fusion operator	Fused opinion	Projection
Belief Constraint Fusion (BCF)	SBoolean(0.303, 0.687, 0.010, 0.579)	0.309
Consensus & Compromise Fusion (CCF)	SBoolean(0.494, 0.489, 0.017, 0.579)	0.503
Aleatory Cumulative Belief Fusion (ACBF)	SBoolean(0.470, 0.490, 0.040, 0.579)	0.493
Epistemic Cumulative Belief Fusion (ECBF)	SBoolean(0.000, 0.149, 0.852, 0.579)	0.493
Average Belief Fusion (ABF)	SBoolean(0.435, 0.454, 0.111, 0.579)	0.499
Weighted Belief Fusion (WBF)	SBoolean(0.453, 0.454, 0.093, 0.579)	0.507

Table 2. Results of applying the fusion operators on the opinions about the value of slot ac0n of bedroom1.

Tool Support. Profile + Plugin

- MagicDraw

The screenshot displays the MagicDraw software interface. The main workspace shows a UML diagram with several elements:

- A central green box representing an `«UncertainElementFusion»` with the text: `beliefs = a_Ziri, b_Ziri`. Below this, it lists various belief functions: `aleatoryCumulativeBF = "SBoolean(0.67, 0.14, 0.18, 0.25)"`, `averagingBF = "SBoolean(0.57, 0.12, 0.31, 0.25)"`, `beliefConstraintFusion = "SBoolean(0.68, 0.15, 0.16, 0.25)"`, `consensusCompromiseBF = "SBoolean(0.65, 0.13, 0.22, 0.25)"`, `epistemicCumulativeBF = "SBoolean(0.62, 0.00, 0.38, 0.25)"`, and `weightedBF = "SBoolean(0.64, 0.11, 0.25, 0.25)"`.
- A box on the left representing an `«UncertainElement»` and `«UncertainElementFusion»` named `Ziri : Person` with the text: `name = "Ziri b Manad al-Sinhay"`.
- Two boxes below representing `«Agent»` instances: `Anne : Geographer` and `Brandon : Librarian`.
- Two boxes below representing `«Belief»` instances: `a Ziri : Belief` (with `agent = Anne` and `opinion = "SBoolean(0.1, 0.2, 0.7, 0.25)"`) and `b Ziri : Belief` (with `agent = Brandon` and `opinion = "SBoolean(0.7, 0.1, 0.2, 0.25)"`).

A context menu is open on the right side of the diagram, listing various actions and their keyboard shortcuts:

Specification	Enter
Diagram Properties...	Shift+Enter
Select in Containment Tree	Alt+B
Go To	>
Display	>
Related Elements	>
Refactor	>
Tools	>
Stereotype	
Complete	
Fuse available opinions	Alt+Shift+F



Conclusión

- Different kinds of uncertainty
- Need to capture, represent and operate with it
- Our contributions to
 - Measurement uncertainty
 - Belief uncertainty
- Future work
 - Empirically validate the usability and usefulness of our contributions with users in real contexts
 - Assist users use the proposed methods / techniques
 - UML/OCL might not be suitable for all users → explore different notations such as tailored DSLs, easier to adapt to the vocabulary and technical knowledge of specific user communities



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