Uncertainty in Software Models

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(part of joint work with more people)

Montreal, July 22, 2022

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Uncertainty

**Uncertainty**: Quality or state that involves imperfect and/or unknown information

- It applies to: predictions of future events, estimations, physical measurements, or properties of a system, its elements or its environment
- due to:
  - Underspecification of the problem or solution domains
  - Lack of knowledge of the system, its environment, or its underlying physics
  - Lack of precision in measurements
  - Imperfect, incorrect, or missing information
  - Numerical approximations
  - Values and parameters indeterminacy
  - Different interpretations of the same evidences by separate parties

“There is nothing certain, but the uncertain” (proverb)
Uncertainty in mechanical systems
Many different formalisms and theories to quantify uncertainty

- Bayesian Belief Networks (BBN)
- Monte Carlo simulations
- Decision theory/trees
- Probabilities
- Fuzzy Logic
- ...
A classification of uncertainty (according to its nature)

- **Aleatory** Uncertainty – A kind of uncertainty that refers to the *inherent* uncertainty due to the *probabilistic variability or randomness* of a phenomenon
  - Examples: measuring the speed of a car, or the duration of a software development process
  - This type of uncertainty is *irreducible*, in that there will always be variability in the underlying variables.

- **Epistemic** Uncertainty – A kind of uncertainty that refers to *the lack of knowledge* we may have about the system (modeled or real).
  - Examples: Ambiguous or imprecise requirements about the expected system functionality, its envisioned operating environment, etc.
  - This type of uncertainty is *reducible*, in that additional information or knowledge may reduce it.

Types of uncertainty (according to their sources)

- **Measurement uncertainty**: A kind of *aleatory* uncertainty that refers to a set of possible states or outcomes of a measurement, where probabilities are assigned to each possible state or outcome.

- **Occurrence uncertainty**: A kind of *epistemic* uncertainty that refers to the degree of belief that we have on the actual existence of an entity, i.e., the real entity that a model element represents.

- **Belief uncertainty**: A kind of *epistemic* uncertainty in which a *belief agent* is uncertain about any of the *statements* made about the system or its environment.

- **Design uncertainty**: A kind of *epistemic* uncertainty that refers to a set of possible design decisions or options, where probabilities are assigned to each decision or option.

- **Environment uncertainty**: Lack of certainty about the surroundings, boundaries and usages of a system and of its elements.

- **Spatiotemporal uncertainty**: Lack of certainty about the geographical or physical location of a system, its elements or its environment, or about the time properties expressed in a statement about the system or its environment.

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Measurement Uncertainty

![Probability distribution with median, mode, and mean highlighted.](image)
Measurement uncertainty

- **Measurement uncertainty**: A kind of *aleatory* uncertainty that refers to a set of possible states or outcomes of a measurement.

- Normally expressed by a parameter, associated with the result of a measurement $x$, that characterizes the dispersion of the values that could reasonably be attributed to the measurand: the standard deviation $u$ of the possible variation of the values of $x$.

- Representation: $x \pm u$ or $(x, u)$.

- **Examples**:
  - Normal distribution: $(x, \sigma)$ with mean $x$, and standard deviation $\sigma$.
  - Interval $[a, b]$: Uniform distribution is assumed $\quad (x, u)$ with $x = \frac{a+b}{2}, \; u = \frac{(b-a)}{2\sqrt{3}}$.

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However, the situation is not the same in software models.
Some problems with Measurement Uncertainty

- Computations with uncertain values must respect the *propagation of uncertainty* (uncertainty analysis)
  - In general, this is a complex problem, which cannot be manually managed

- *Comparison of uncertain values* is no longer a Boolean property!
  - How to compare $17.7 \pm 0.2$ with $17.8 \pm 0.2$?

- Other primitive datatypes are also affected by uncertainty
  - Strings (OCR)
  - Enumerations
  - Collections
UBooleans are pairs (b, c)
- where b:Boolean and c:Real, c ∈ [0, 1]
- c represents the **confidence** that the actual value of the value is indeed b
- Canonical form: (true, c)
- Equivalence relation: (b, c) = (not b, 1 - c)

**Operations**
- Redefined basic operations: and, or, not
- Redefined secondary operations: implies, equivalent, xor
- Conversion operations: toBoolean() and toBooleanC(c:Real)
Primitive datatypes extended with Uncertainty

- **Extended primitive datatypes**
  - Real → UReal  UReal(17.8, 0.2) ≡ 17.8 ± 0.2
  - Boolean → UBoolean  UBoolean(true, 0.8)
  - String → UString  UString("Implementaci6n", 0.93)
  - Enum → UEnum  UColor{ (#red, .9), (#orange, 0.09), (#purple, 0.01) }

- An algebra of operations on uncertain datatypes extending OCL/UML types
- Operations are **closed** in this algebra and automatically **propagate** uncertainty

```plaintext
use> ?UReal(17.7, 0.2) < UReal(17.8, 0.2)  -> UBoolean(true, 0.1974125487) : UBoolean
use> ?UReal(17.7, 0.2) = UReal(17.8, 0.2)  -> UBoolean(true, 0.8025874513) : UBoolean
use> ?UReal(17.7, 0.2) > UReal(17.8, 0.2)  -> UBoolean(true, 0.0) : UBoolean
```

Belief Uncertainty

propositions

well formed  ill-formed

good guesses  nonsense

true  false

beliefs

knowledge

justified

denial  lucky denial

false positives

(CC-BY 4.0) 2014 Ryan Reece philosophy-in-figures.tumblr.com
Belief uncertainty

- **Belief uncertainty**: A kind of *epistemic* uncertainty in which the modeler, or any other *belief agent*, is uncertain about any of the *statements* made about the system or its environment.
  - By nature, it is always *subjective*
  - *It may not always be possible to determine whether or not a belief statement is valid.*
    - A belief statement may not necessarily correspond to objective reality.
    - This means that it could be completely false, or only partially true, or completely true.
  - The validity of a statement may only be *meaningfully defined within a given context or purpose*.
    - Thus, the statement that “the Earth can be represented as a perfect sphere” may be perfectly valid for some purposes but invalid or only partly valid for others.

- **Belief agent**: An entity (human, institution, even a machine) that holds one or more beliefs

- **Belief statement**: *Statement* qualified by a *degree of belief*

- **Degree of belief**: Confidence assigned to a statement by a belief agent. Normally expressed by quantitative or qualitative methods (e.g., a grade or a probability “credence”)

Classical Boolean Logic

\[ \text{false} = 0 \]
\[ \overline{X} \text{ vertex (disbelief)} \]

\[ \text{true} = 1 \]
\[ X \text{ vertex (belief)} \]
false = 0

\( \overline{X} \) vertex (disbelief)

true = 1

\( X \) vertex (belief)
Kleene (three-valued) Logic

Uncertainty

\[ X \text{ vertex (disbelief)} \quad \bullet \quad X \text{ vertex (belief)} \]
Subjective Logic

\[ (0,0,1) \]

\[ U \text{ vertex (uncertainty)} \]

\[ (0,1,0) \]

\[ X \text{ vertex (belief)} \]

\[ (1,0,0) \]

\[ \bar{X} \text{ vertex (disbelief)} \]

\[ b_X \]

\[ d_X \]

\[ \omega_X \]

\[ u_X \]

\[ P_X \]

\[ a_X \]
Subjective logic in UML/OCL

- **SBoolean**(*b*,*d*,*u*,*a*)
  - *b* represents the degree of **belief** that the agent has about the statement
  - *d* represents the degree of **disbelief**
  - *u* represents the **uncertainty** that the agent expressing the opinion has about the statement, i.e., the degree of trust
  - *a* is the (objective) **prior probability** assigned to the statement, based on previous evidence (also called “**base rate**”).

- \[ b + d + u = 1 \]

- Boolean values lifted to SBoolean:
  - true ≡ SBoolean(1, 0, 0, 1)
  - false ≡ SBoolean(0, 1, 0, 0)

- UBoolean values lifted to SBoolean:
  - Uboolean(true,c) = SBoolean(c,1-c,0,c)

- Projection from SBoolean to UBoolean:
  - SBoolean(*b*,*d*,*u*,*a*).projection() = UBoolean(true, *b* + *u* *a)
Belief Profile
Belief Profile

![Diagram of a belief profile with classes and relationships]

**SmartHouse**
- `+house` (1)
- `+room` (0..*)

**Room**
- `+acOn : UBoolean`
- `+temperature : UReal`
- `+humidity : UReal`

**bedroom1 : Room**
- `acOn = UBoolean(true, 0.58)`
- `humidity = UReal(79.0, 1.0)`
- `temperature = UReal(25.0, 0.5)`

**Agents and Beliefs**
- `h : SmartHouse`
- Ada : Person
- Bob : Person
- Cam : Person

**Beliefs**
- `ada_temp : Belief
  agent = Ada
  opinion = SBoolean(0.95, 0.0, 0.05, 0.5)`
- `bob_temp : Belief
  agent = Bob
  opinion = SBoolean(0.1, 0.8, 0.1, 0.5)`
- `cam_temp : Belief
  agent = Cam
  opinion = SBoolean(0.2, 0.2, 0.6, 0.5)`
- `ada_humidity : Belief
  agent = Ada
  opinion = SBoolean(0.75, 0.15, 0.1, 0.5)`
- `bob_humidity : Belief
  agent = Bob
  opinion = SBoolean(0.5, 0.1, 0.4, 0.5)`
- `cam_humidity : Belief
  agent = Cam
  opinion = SBoolean(0.3, 0.3, 0.4, 0.5)`
Fusion Operators

- Merge the beliefs of different agents on the same model elements

<table>
<thead>
<tr>
<th>Belief Constraint Fusion (BCF)</th>
<th>Willingness to compromise</th>
<th>Assumed dependency between opinions</th>
<th>Vacuous opinion as neutral element</th>
<th>Preserve shared beliefs, and conflicting opinions are turned into vague belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO. If conflict, undefined result.</td>
<td>Only for fusion of agents' preferences</td>
<td>YES</td>
<td>NO</td>
<td></td>
</tr>
<tr>
<td>Cumulative Belief Fusion (CBF)</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Averaging Belief Fusion (ABF)</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Weighted Belief Fusion (WBF)</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Consensus and Compromise Fusion (CCF)</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

[1] Not applicable for fusing evidence from different agents to determine the most likely hypothesis or actual event.
[2] There are two types of Cumulative Belief Fusion operators: Epistemic Cumulative Fusion (E-CBF) and Aleatory Cumulative Fusion (A-CBF). Their use depends on the nature of the fused opinions.
Fusion Operators

<table>
<thead>
<tr>
<th>Fusion operator</th>
<th>Fused opinion</th>
<th>Projection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belief Constraint Fusion (BCF)</td>
<td>SBoolean(0.303, 0.687, 0.010, 0.579)</td>
<td>0.309</td>
</tr>
<tr>
<td>Consensus &amp; Compromise Fusion (CCF)</td>
<td>SBoolean(0.494, 0.489, 0.017, 0.579)</td>
<td>0.503</td>
</tr>
<tr>
<td>Aleatory Cumulative Belief Fusion (ACBF)</td>
<td>SBoolean(0.470, 0.490, 0.040, 0.579)</td>
<td>0.493</td>
</tr>
<tr>
<td>Epistemic Cumulative Belief Fusion (ECBF)</td>
<td>SBoolean(0.000, 0.149, 0.852, 0.579)</td>
<td>0.493</td>
</tr>
<tr>
<td>Average Belief Fusion (ABF)</td>
<td>SBoolean(0.435, 0.454, 0.111, 0.579)</td>
<td>0.499</td>
</tr>
<tr>
<td>Weighted Belief Fusion (WBF)</td>
<td>SBoolean(0.453, 0.454, 0.093, 0.579)</td>
<td>0.507</td>
</tr>
</tbody>
</table>

Table 2. Results of applying the fusion operators on the opinions about the value of slot ac0n of bedroom1.
Tool Support. Profile + Plugin

- MagicDraw

![MagicDraw Diagram](image-url)
Conclusión

- Different kinds of uncertainty

- Need to capture, represent and operate with it

- Our contributions to
  - Measurement uncertainty
  - Belief uncertainty

- Future work
  - Empirically validate the usability and usefulness of our contributions with users in real contexts
  - Assist users use the proposed methods / techniques
  - UML/OCL might not be suitable for all users → explore different notations such as tailored DSLs, easier to adapt to the vocabulary and technical knowledge of specific user communities
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